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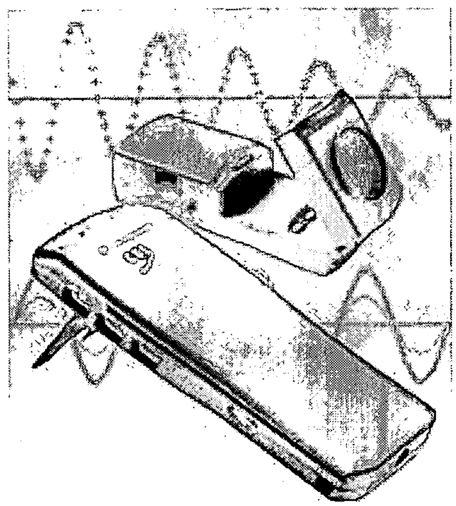
ABSTRACT

This paper presents three activities on how to analyze rates of change in real-life situations using TI-83 calculators and computer-based laboratories. Activities include 24 hour temperature data, the temperature of a light bulb, and an M&M toss. Each section contains descriptions of equipment/materials, data collection, and data analysis. The document ends with "Families of Functions" activities. (YDS)



UNDERSTANDING RATES OF CHANGE

TEACHERS TEACHING WITH TECHOLOGY INTERNATIONAL CONFERENCE

COLUMBUS, OHIO
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ANALYZING RATES OF CHANGE IN REAL-LIFE SITUATIONS

24-HOUR TEMPERATURE DATA ACTIVITY

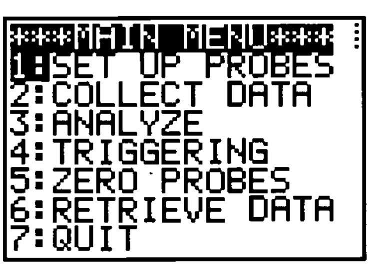
The purpose of this experiment is to collect temperature data over a 24-hour period.

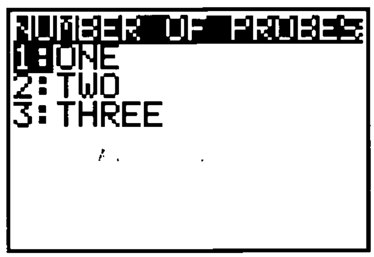
EQUIPMENT: TI-83Plus, CBL, temperature probe, PHYSICS** program or application, link cord.

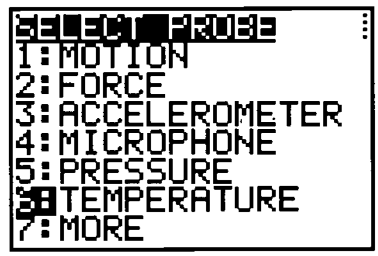
DATA COLLECTION:
Suggestion: Insert new batteries into the CBL or use the electrical adapter for your ViewScreen.

1. Securely join the TI-83 Plus and the CBL with the black link cord. Turn on the CBL.

2. Execute the Physics Program or Physics Application (Vernier). When the first screen below appears, choose **1:SET UP PROBES**. Then choose **1:ONE** for the number of probes. On the next screen, choose **6: TEMPERATURE.**





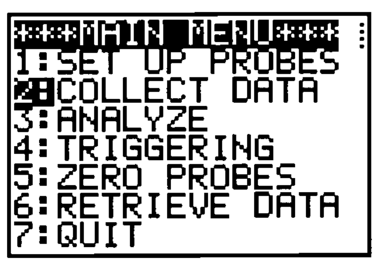


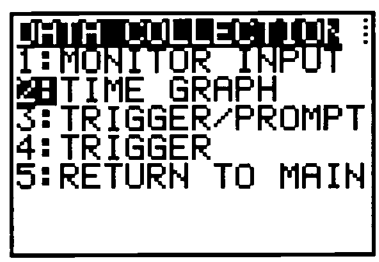
3. Plug the temperature probe into Channel 1, as directed.

4. Choose **1:USED STORED** on the Calibration submenu.

5. On the Main Menu Screen, choose **2:COLLECT DATA.**

6. Choose **2:TIME GRAPH** (see below):

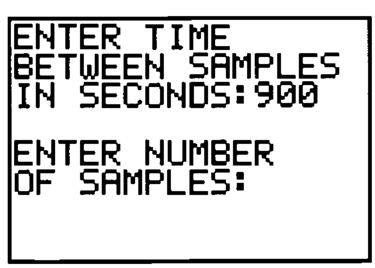


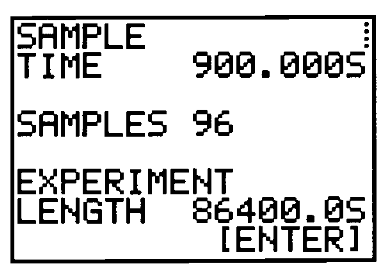


7. Enter **900** for ENTER TIME BETWEEN SAMPLES IN SECONDS. (15 minutes). Press ⊆.

8. Enter **96** for ENTER NUMBER OF SAMPLES. (24 hours) Press ⊆.

9. Check your entries on the next screen and press ⊆.

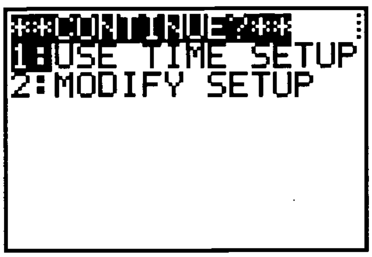


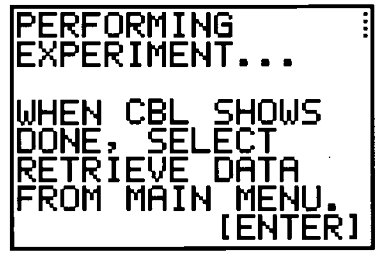


10. Choose **1:USE TIME SETUP** on the Continue Menu if the information is correct. If there is an error, choose **2:MODIFY SETUP** and repeat Steps 7 – 9.

11. When you have positioned your equipment, press ⊆ to BEGIN DATA COLLECTION.

12. The PERFORMING EXPERIMENT screen will appear.

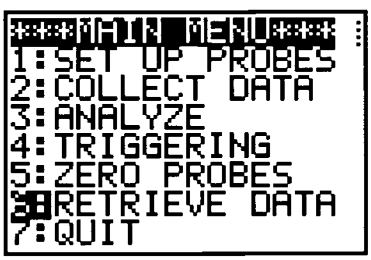


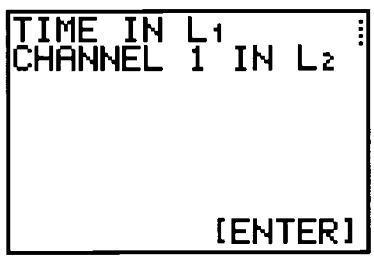


13. The next day, turn on the sameTI-83Plus and re-execute the Physics Program or Application. Choose **6:RETRIEVE DATA** from the Main Menu.

14. Make sure than the CBL shows DONE and press ⊆.

15. The program will indicate that TIME is in L1 and TEMPERATURE is in L2.





16. When you press ⊆ again, the graph of TEMPERATURE vs. TIME will appear.

DATA ANALYSIS

1. At what time was the temperature the highest? The lowest?

2. What was the average temperature for the 24 hour period?

3. Were there any times when the temperature stayed fairly constant?

4. At what times was the temperature rising? Falling?

5. When did the greatest rate of change occur? When was the temperature rising most rapidly? When was the temperature falling most rapidly?

6. What was the average rate of change in temperature (include units) between midnight and 6 am? Between midnight and 3 am? Between midnight and 1 am? Between midnight and 12:30am? Between midnight and 12:15 am?
   Approximate the rate of change of the temperature at exactly midnight. (Very informal introduction to instantaneous rate of change).

**PHYSICS program or application available at : http://www.vernier.com/legacy/cbl/progs.html

TEMPERATURE OF A LIGHT BULB

The purpose of this experiment is to record the temperature of a light bulb for the first 10 seconds after it is turned on.

EQUIPMENT: TI-83Plus, CBL, temperature probe, PHYSICS program or application, link cord, light bulb, electrical socket.

DATA COLLECTION:

1. Securely join the TI-83 Plus and the CBL with the black link cord. Turn on the CBL. Insert the temperature probe into Channel 1.
2. Execute the Physics Program or Physics Application (Vernier).
3. Choose the TEMPERATURE probe.
4. Choose TIME GRAPH with TIME BETWEEN SAMPLES as .1 seconds and NUMBER OF SAMPLES as 100.
5. Place the temperature probe on the light bulb and press ⊆ to start the data collection.

DATA ANALYSIS:

1. Compute the average rates of change of the temperature from 0 to 2 seconds, from 2 to 4 seconds, from 5 to 8 seconds, from 5 to 10 seconds, and from 0 to 10 seconds. (Include units.)
2. What do you notice about all these rates of change?
3. What was the initial temperature of the probe when the light bulb was turned on.?
4. Find an equation which fits the temperature data well. y = ______________________________
5. Enter your equation into Y1 and compare the graphs. Make necessary adjustments to your equation.
6. Use your equation to estimate the temperature of the bulb 25 seconds after it is turned on. Explain.
7. Can you use your equation to estimate the temperature of the bulb 5 minutes after it is turned on? Explain.
8. If you can only keep your finger on the bulb until it reaches 130°F, how much time will elapse before you need to remove your finger?
9. Store L1 in L3, by pressing ψ ℵ ↵ ψ ℜ
10. Go to L3 and delete the first entry in L3, by pressing ⊆; then ~ to the first entry in L3 and press {.
11. Compute delta(L2)/delta(L1):
    ~ to the label at the top of L4 and press ψ , ~ over to OPS and press ← ψ ℑ / ∞.
    Repeat the process again: ψ , ~ over to OPS and press ← ψ ℵ / ⊆.
12. Look at the data in L4. What do you notice about those values?
13. Turn on a StatPlot of L4 (y-axis) vs. L3 (x-axis). (Rate of change vs. Time).
    What are the units on the y-axis?
    What are the units on the x-axis?
    Trace on the StatPlot. What do you notice about the y-values?
    How do those values fit with the equation that you found?

M & M TOSS ACTIVITY

In this experiment, you will toss m & m's from a styro cup and count the number of m & m's with the **m** showing upward. You may eliminate (in any way you wish) the m & m's which are **m down**. Then take the remaining m & m's, toss and count the **m's up** again. Repeat the process until all the m & m's are eliminated. You will then find an equation which fits your data. You will also analyze the rate of change function.

MATERIALS: about 80 m & m's per group, 2 styro cups per group, 1 plastic sandwich bag for each student (proper m & m hygiene)

DATA COLLECTION:

1. Count the number of m & m's you have in your cup. Make sure they all have an **m** on one side. If you have any blanks, dispose of them and do not count them. Enter the total number in the **NUMBER OF m & m's UP** column below for **TOSS #0.** (Chart is located on the back side of this sheet.) Leave the **NUMBER OF m & m's DOWN** column empty for Toss #0.

2. **TOSS #1**
   Place the m & m's in the styro cup and shake them up well. Pour them out on the desk and count the number of m & m's with the **m up**. Record this number in the **NUMBER OF m & m's UP** for **TOSS #1**. Count the number of m & m's which do not have an m up, record that number in the **NUMBER OF m & m's DOWN** column, and then remove those m & m's by any method you prefer.

3. **TOSS #2**
   Place all the remaining m & m's in the styro cup again. Repeat the shaking, pouring and counting m's up processes. Record the number in the **NUMBER OF m & m's UP** column and the **NUMBER OF m & m's DOWN** column below for **TOSS #2**.

4. Repeat the whole process until all the m & m's are gone.

DATA ANALYSIS:

1. Enter **TOSS NUMBERS** in L1 and the **NUMBER OF m & m's UP** in L2. Enter the **TOSS NUMBERS** in L3 omit **TOSS #0** this time. Enter the **NUMBER OF m & m's DOWN** in L4.

2. Turn on a StatPlot of L2 (y-axis) vs. L1 (x-axis) (NUMBER OF m & m's UP vs. TOSS #)

3. Can you find the equation of a LINE which fits your data WELL? Why or why not?

4. Examine the situation from a theoretical standpoint:

   How many m & m's did you start with? ________________

   On the first toss, how many m & m's would you **EXPECT** to be **m up**? ________

   On the second toss, how many m & m's would you **EXPECT** to be **m up**? ________

   On the third toss, how many m & m's would you **EXPECT** to be **m up**? ________

   On the sixth toss, how many m & m's would you **EXPECT** to be **m up**? ________

   On the tenth toss, how many m & m's would you **EXPECT** to be **m up**? ________

   On the **xth** toss, how many m & m's would you **EXPECT** to be **m up**? ________

5. Write an equation (not linear) which describes the relationship between the **NUMBER OF m & m's UP (y)** and the **TOSS NUMBER (x)** for your data.

y = ____

6. Generate an x/y chart below using your equation in #5. (Chart at right below.)

7. Enter the equation from #4 in Y1.

8. How well does the equation fit the data you collected experimentally? How close are the two graphs?

EXPERIMENTAL DATA:

TOSS #	NUMBER OF m&m's UP	NUMBER OF M & m's DOWN
O		
1		
2		
3		
4		
5		
6		
7		
8		

EQUATION PREDICTIONS:
(See next page for chart)

ANALYZING THE RATE OF CHANGE FUNCTION:

9. Examine the entries in the **NUMBER OF m & m's DOWN** COLUMN. This column represents the ***DECREASE—THE RATE OF CHANGE*** **IN THE NUMBER OF m & m's UP.** By examining the numbers in that column, what conclusions can you draw about the rate at which the m & m function is changing?

10. Do you expect the rate of change function to be linear? Explain why or why not.

11. Turn on a StatPlot of L4 (y-axis) vs. L3 (x-axis). (AMOUNT OF CHANGE vs. TOSS NUMBER)

12. What are the units on the y-axis? On the x-axis?

13. Which family of functions does the rate of change function appear to belong to?

TOSS #	NUMBER OF m&m's UP	NUMBER OF M & m's DOWN
O		
1		
2		
3		
4		
5		
6		
7		
8		

SUGGESTIONS FOR OTHER DATA ANALYSIS ACTIVITIES:

Possible areas of emphasis – depending on the grade and level of math class:
- Graph of original data.
- Identifying the parent function type which best fits the original data.
- Answering practical questions related to the data.
- Graph of the rate of change function.
- Units of the rate of change function.
- Identifying the parent function type of the rate of change function.
- Making a connection between the parent function type and the rate of change function type.
- Making connections between the graphs of the parent function and the rate of change function:
  - Original data increasing ? rate of change function is positive.
  - Original data decreasing ? rate of change function is negative.

1. Daily/weekly/monthly changes of the Dow Jones Industrial Index.
2. Weights of pennies (pre-1983, post-1983).
3. Population: World/US/City. – Good Internet Search problems.
4. Height of amaryllis plants. (Logistic function.)
5. Number of cars on the school parking lot from 30 minutes before school begins to 30 minutes after school begins. Summarize data in 5 minute intervals. (Logistic function.)
6. POOL CBL Activity – Real World Math Book
7. Weight of a water bottle as it drains.
8. Rate of change (velocity) of a ball as it bounces; Speed of a ball as it bounces.
9. Any type of data which interests the students or which they can use/have used for a report in another class.

FAMILIES OF FUNCTIONS ACTIVITIES

1. PARENT FUNCTION CALISTHENICS

2. HUMAN GRAPHING

Use adding machine paper to mark off 2 sets of coordinate axes on the floor. (Make each unit approximately 2 feet in width.)

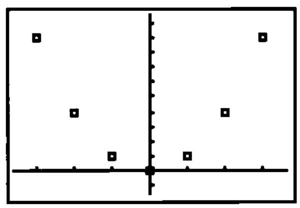
Divide the class into groups of 7students. Assign each student an integer from –3 to +3, inclusive.
(For the trig functions, you will need groups of 9 students and assign values of - , -3 /4, - /2, - /4, 0, /4, /2, 3 /4, .)

Then have each group pair up with another group. One group will model the parent function and the other will model the rate of change function.

Call off the equation of a parent function and have the parent function groups arrange themselves to form the graph of that equation.

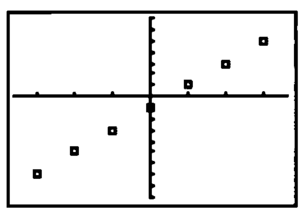
For example:

$y = x^2$



After the equation is modeled correctly, have the rate of change group stand on the x axis of the second adding machine coordinate system.

Beginning with $x = -3$, each student from the original function group calls out how many units her y-value has increased or decreased from the student on her left. Then the $x = -3$ student in the rate of change group should move off the x-axis to that y-coordinate. Then repeat the process with $x = -2, -1, 0, 1, 2, 3$.



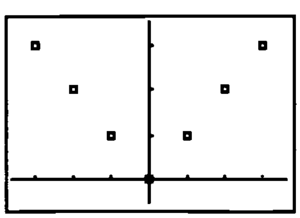
Which parent function type would seem to be a good model for the rate of change function?

(Don't have them find the actual equations of the rate of change functions is you are trying to make an immediate connection to the derivative. You are working with the <u>average</u> rate of change here, not the <u>instantaneous</u> rate of change.)

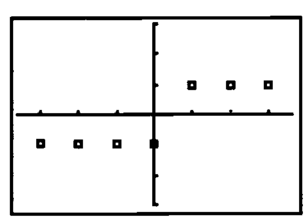
Do you see any patterns in the amounts of change? Is the actual amount of change constant? Increasing? Decreasing?

<u>2ND Example:</u> <u>$y = |x|$</u>

Parent function:



Rate of change function:



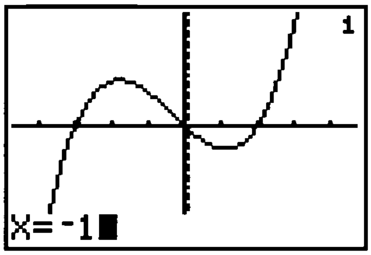
3. USING dy/dx TO VISUALIZE THE *INSTANTANEOUS* RATE OF CHANGE.

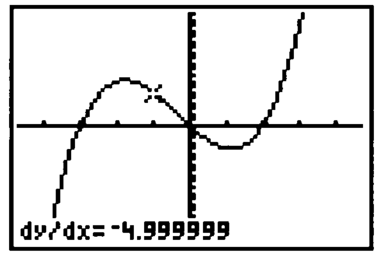
Enter an equation into Y1. Ex: Y1 = $x^3 + x^2 - 6x$. Set an appropriate window so that a complete graph is visible.

Use the CALC Menu to compute the rate of change at a particular x-value:

Press ψ ρ and to ÷ and press ⊆.

Type in the x-value where you want to find the rate of change. Then press ⊆ again.





Then ask the students what dy/dx = -5 really means. "When x = -1, y is decreasing by 5 units." Be sure to note x values where horizontal tangents would occur.

Students can calculate the rate of change at several integral values for x and then graph the RATE OF CHANGE vs. the X.

PARENT FUNCTIONS & RATE OF CHANGE FUNCTIONS – PAGE 1

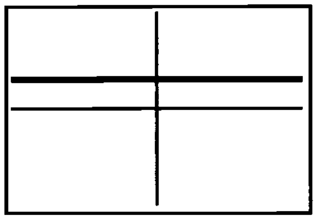
FUNCTION TYPE | ***GRAPH*** | ***RATE OF CHANGE FUNC.***

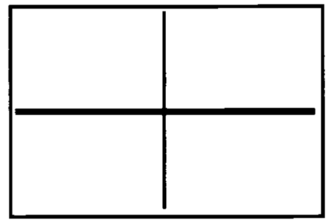
CONSTANT

y = k

Domain: all Reals

Range: k



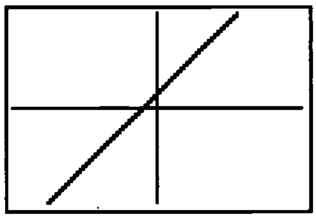


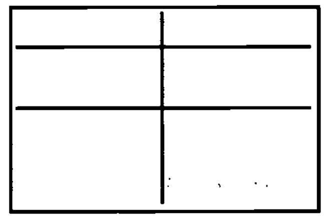
LINEAR

y = mx + b

Domain: all Reals

Range: all Reals



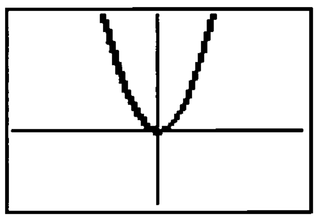


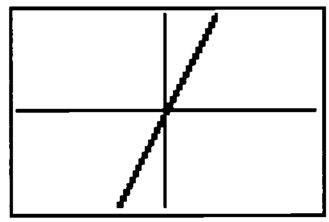
QUADRATIC

y = x^2

Domain: all Reals

Range: y 0



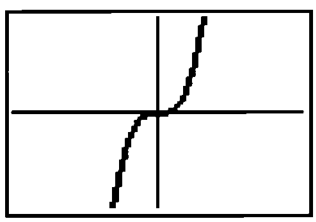


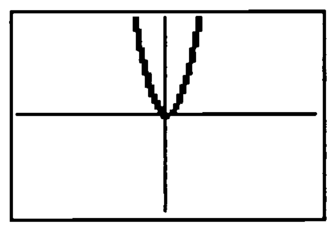
CUBIC

y = x^3

Domain: all Reals

Range: all Reals



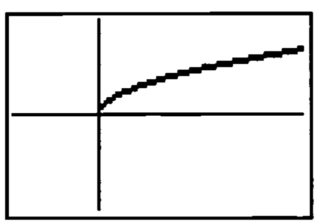


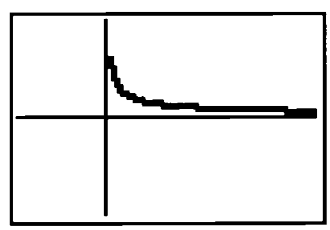
SQUARE ROOT

y = ♣x

Domain: x 0

Range: y 0



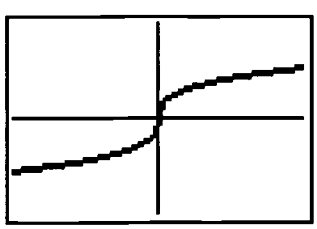


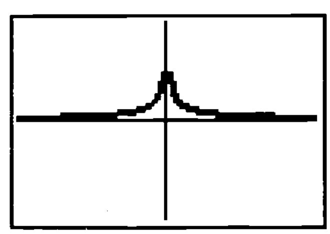
CUBE ROOT

y = ^{3}♣x

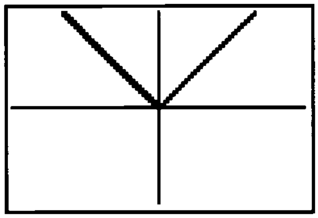
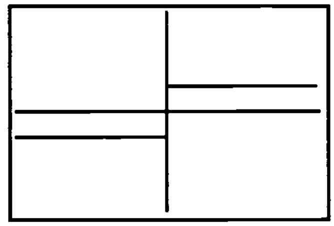
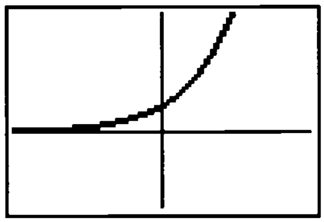
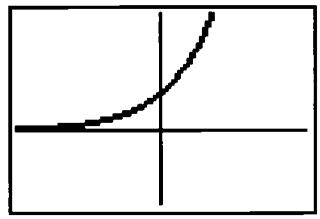
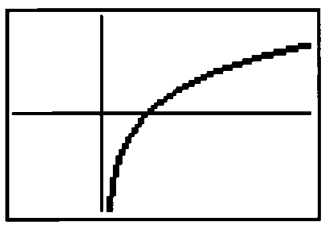
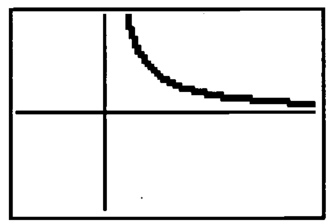
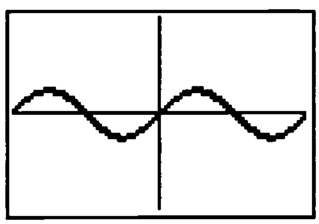
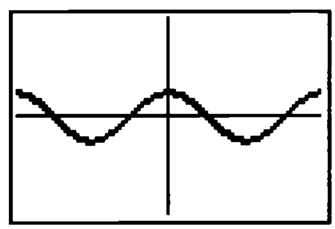
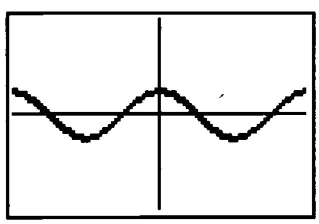
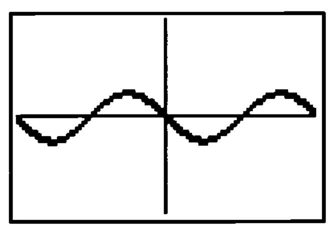
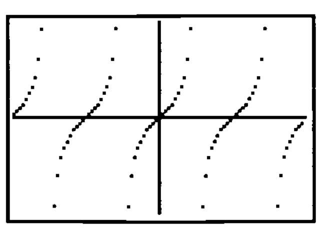
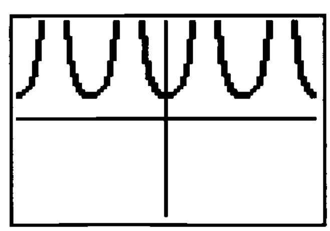
Domain: all Reals

Range: all Reals





PAGE 2

FUNCTION TYPE	***GRAPH***	***RATE OF CHANGE***
ABSOLUTE VALUE $y = \lvert x \rvert$ Domain: all Reals Range: y 0		
EXPONENTIAL $y = b^x, b > 0$ Domain: all Reals Range: $y > 0$		
LOGARITHMIC $y = \log_b x, b > 0$ Domain: $x > 0$ Range: all Reals		
SINE $y = \sin x$ Domain: all Reals Range: -1 y 1		
COSINE $y = \cos x$ Domain: all Reals Range: -1 y 1		
TANGENT $y = \tan x$ Domain: Range: all Reals		

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